P.119: 4

$$\int_0^{\pi} e^{(1+i)x}\, dx = \left[\frac{e^{(1+i)x}}{1+i} \right]_0^{\pi} = \frac{e^{(1+i)\pi} - 1}{1+i}$$

$$= \frac{e^{\pi}(\cos\pi + i\sin\pi) - 1}{1+i} \cdot \frac{1-i}{1-i}$$

$$= \underbrace{-\frac{e^{\pi}+1}{2}}_{Re} + i\, \underbrace{\frac{e^{\pi}+1}{2}}_{Im}$$

P.124: 5

Following the suggestion, the Cauchy - Riemann eqs

are $u_x = v_y$ and $u_y = -v_x$

Substitude and we obtain

$$w'(t) = \underbrace{(u_x + i v_x)}_{f'[z(t)]}\, \underbrace{(x' + i y')}_{z'(t)} \qquad \text{(Verify!)}$$

P.133: 3

Tracing counterclockwisely, the integrals of 4 segs

are $\left[e^{\pi x} \right]_0^1 + \left[-e^{(1-iy)\pi} \right]_0^1 + \left[e^{\pi x} \right]_0^1 + \left[e^{-i\pi y} \right]_0^1$

$$= (e^{\pi + x} - 1) + 2e^{\pi} + (e^{\pi} - 1) - 2$$

$$= 4(e^{\pi} - 1)$$

4: $\int_C f\,dz = \int_{-1}^{0} 1 + i\cdot 3x^2\,dx + \int_0^1 4x^3(1+i\cdot 3x^2)\,dx$

$= \left[x + ix^3\right]_{-1}^{0} + \left[x^4 + i\cdot 2x^6\right]_0^1$

$= (1+i) + (1+2i)$

6: $\int_C f\,dz = \int_0^{\pi} (e^{it})^{i}\cdot ie^{it}\,dt$

$= \frac{i}{i-1}\left[e^{(i-1)t}\right]_0^{\pi}$

$= -\frac{1+e^{-\pi}}{2}\cdot(1-i)$

P.138: 2

The ~~seg~~ length of seg $L = \sqrt{2}$

~~For~~ Since $d = \frac{\sqrt{2}}{2}$, we have all z on the seg

that $1/|z|^4 \le d^{-4} = 4 = M$.

So, $\left|\int_C \frac{dz}{z^4}\right| \le ML = 4\sqrt{2}$

3: The length $L = 3 + 5 + 4 = 12$

For $x \leq 0$, $|e^z - \bar{z}| \leq e^x + \sqrt{x^2+y^2}$

$\leq 1 + 4 = 5$

Thus, $\int_C \cdots dz \leq ML = 60$

5: $|\text{Log } z| = |\ln R + i\theta| \leq \ln R + \pi$

$$\left| \int_{C_R} \frac{\log z}{z^2} dz \right| \leq \frac{\ln R + \pi}{R^2} \cdot 2\pi R$$

$$= 2\pi \left(\frac{\pi + \ln R}{R} \right)$$

which is of the form $\frac{\infty}{\infty}$, as $R \to \infty$.

By LR, $\lim_R \frac{\pi + \ln R}{R} = \lim_R \frac{R^{-1}}{1} = 0$

P. 147: 5

Let $F(z) = \frac{z^{i+1}}{i+1} = \frac{e^{(i+1)(\ln r + i\theta)}}{i+1}$, $r > 0,\ -\frac{\pi}{2} < \theta < \frac{3}{2}\pi$

Then $F'(z) = f(z) = z^i$

With simple calculation, $F(-1) = -\frac{e^{-\pi}}{2}(1-i)$, and $F(1) = \frac{1-i}{2}$

$$\int_{-1}^{1} f(z)\, dz = [F(z)]_{-1}^{1} = \frac{1+e^{-\pi}}{2}(1-i)$$

P.159: 4

a) Lower horizontal: $\int_{-a}^{a} e^{-x^2}\,dx = 2\int_0^a e^{-x^2}\,dx$

Upper: $\int_{a}^{-a} e^{-(x+ib)^2}\,dx$

$$= -e^{b^2}\int_{-a}^{a} e^{-x^2}\left(\cos(-2xb) + i\sin(-2xb)\right)dx$$

$$= -2e^{b^2}\int_0^a e^{-x^2}\cos(2bx)\,dx$$

Left: $\int_b^0 e^{-(-a+yi)^2}\,i\,dy = -ie^{-a^2}\int_0^b e^{y^2}e^{2ayi}\,dy$

Right: $\int_0^b e^{-(a+yi)^2}\,i\,dy = ie^{-a^2}\int_0^b e^{y^2}e^{-2ayi}\,dy$

By C-G thm, the 4 integrals sum to zero. i.e.

$$\int_0^a e^{-x^2}\cos(2bx)\,dx = e^{-b^2}\int_0^a e^{-x^2}\,dx + \frac{i}{2}e^{-(a^2+b^2)}.$$

$$\left(\int_0^b e^{y^2}(\cos(2ay) - i\sin(2ay))\,dy - \int_0^b e^{y^2}(\cos(2ay)+i\sin(2ay))\,dy\right)$$

$$= e^{-b^2}\int_0^a e^{-x^2}\,dx + e^{-(a^2+b^2)}\int_0^b e^{y^2}\sin(2ay)\,dy$$

b) $\lim_a e^{-(a^2+b^2)}\int_0^b e^{y^2}\sin(2ay)\,dy \le \lim_a e^{-(a^2+b^2)}\cdot \int_0^b e^{y^2}\,dy = 0$

Hence, $\int_0^\infty e^{-x^2}\cos(2bx)\,dx = \frac{\sqrt{\pi}}{2}e^{-b^2}$

6: $$\int_C f(z)\,dz = \int_0^1 f(t)\,dt + \int_0^\pi f\left(\tfrac{1}{2}+\tfrac{1}{2}e^{it}\right)\cdot\tfrac{i}{2}e^{it}\,dt$$

$$= \int_0^1 \sqrt{t}\,dt + \int_0^\pi \sqrt{\tfrac{1}{2}+\tfrac{1}{2}e^{it}}\cdot\tfrac{i}{2}e^{it}\,dt$$

$$= \int_0^1 \sqrt{t}\,dt + \int_1^0 \sqrt{u}\,du \qquad \left(\begin{array}{l} u = \tfrac{1}{2}+\tfrac{1}{2}e^{it} \\ du = \tfrac{i}{2}e^{it}\,dt \end{array}\right)$$

$$= 0$$

Note that $\displaystyle\lim_{z\to 0}\frac{f(z)-f(0)}{z-0} = \lim_{r\to 0}\frac{1}{\sqrt{r}\,e^{i\frac{\theta}{2}}} = \infty$

So, f is not analytic at 0, and thus the C-G thm is not applicable. $(0\in C)$